Standard essential patents: who is really holding up (and when)?

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ABSTRACT

This paper analyzes the effect of injunctions on royalty negotiations for standard essential patents. We develop a model in which Courts grant injunctions only when they have sufficient evidence that the prospective licensee is unwilling, in line with the way we understand Courts to operate in Europe. In such a framework the prospective licensee has a powerful strategic tool: the offers that he makes to the patent holder will affect the royalty rate that the Court may adopt as well as the probability of being subject to injunctions (and the liability for litigation costs). We find that despite the availability of injunctions, the holder of a sufficiently weak patent will end up accepting below FRAND rates, in particular when litigation cost are high. We also find that the prospective licensee will sometimes prefer to litigate and the holder of a sufficiently strong patent will always end up in litigation by rejecting offers below FRAND. This arises in particular when the prospective licensee has little to fear from being found unwilling, namely when the trial takes time (so that the threat of injunctions is less powerful), and when litigation costs are low. Importantly, we thus find that hold up (royalties above the fair rate) as well as reverse hold up (royalties below the fair rate) may arise in equilibrium.

Keywords: standard essential patent, injunctions, hold up, reverse hold up
JEL Classification: K41, L49, O34

I. INTRODUCTION

The objective of this paper is to analyze how injunctions, which prohibit the sale of products which make unauthorized use of standard essential patents (SEPs), affect negotiations on royalties for these patents. Disputes regarding royalties for SEPs, non SEPs as well as design rights and the pursuit of injunctions in the context of these disputes have intensified in the last few years. In the mobile device market, litigation has pitched manufacturers (and software developers) such as Apple, Microsoft, Samsung and Motorola against each other, across multiple jurisdictions and before several courts. Against this backdrop, leading competition authorities such as DG Competition and the Federal Trade Commission have embraced the concern that injunctions sought in particular by SEP holders may enable them to extract royalties from licensees that can be qualified as “excessive”. These authorities are concerned about the prospect of "hold-up", an issue that has been acknowledged in the literature for some time.

In particular, Lemley and Shapiro (2007)[1] and Shapiro (2010)[2] recognize that the threat of an injunction can impose a large on-going loss on a prospective licensee who has made a specific investment.[3] This threat will improve

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[1] Mark Lemley, and Carl Shapiro, *Patent Holdup and Royalty Stacking*, 85 Tex. L. R. 2163, 2172-73 (2007)

[2] Carl Shapiro, *Injunctions, Hold-Up, and Patent Royalties*, American Law and Economics Review 12, no. 2 509-557 (2010)

[3] See also Joseph Farrell, John Hayes, Carl Shapiro and Theresa Sullivan, *Standard setting, patents, and hold-up*, Antitrust LJ 74 (2007): 603, which argues that there is a particular risk of an inefficient hold up in the context of SEPs and discuss techniques for avoiding it. The focus of that paper is the so-called patent ambush, a failure to disclose patents essential to standards early, but they also discuss how hold up can arise even when patents were duly disclosed early.

the bargaining position of the patent holder. A licensee who has inadvertently infringed a patent will be held up due to the large costs of circumventing it ex post. As a result, the patent holder will be able to obtain royalties in excess of their 'fair value" – that Shapiro takes as a share of the expected incremental value of the final product which is brought about by the IPR. Even if the licensee is fully aware that his implementation may infringe the patent, he may prefer not to design around the IPR of the patent holder – in particular when the patent is weak. The licensee will then find himself in exactly the same position as if the patent had been inadvertently infringed. Shapiro concludes that the royalties obtained by the patent holder under the threat of an injunction will often exceed the "fair value" that could be determined by a court. A key assumption in Shapiro's framework is that the Court grants an injunction whenever it is requested by the patent holder and the patent is found valid and infringed.

However, in reality, validity is not a sufficient condition for the courts to grant injunctions on SEPs. Indeed, in contrast to other types of IPR, SEP holders are generally expected to commit to license their technology on FRAND terms and Courts will give effect to that commitment. The rationale for this is simple: as the inclusion of a technology in a standard increases its value (a consequence of the network effect that interoperability triggers) and locks in implementers, patent holders would be in a favorable position to hold them up.[4] Commitments to license on FRAND terms are thus meant to reduce the scope for hold up and are commonly understood with reference to the terms that would have been proposed before the technology was included in the standard. In that sense the commitments can be interpreted as ensuring that the patent holder obtains royalties reflecting the contribution of its patented technology to the standard – but only up to the second-best alternative that would obtain ex ante when alternative technologies compete (i.e. when hold up is not a concern). Contrary to what Shapiro assumes, in the case of a disagreement between the prospective licensee and patent holder for SEPs, Courts will thus not merely award an injunction upon a finding of validity and infringement of the patent - injunctions will typically only be awarded if some additional conditions are met.

We model the Court procedure for resolving SEP licensing disputes in line with our understanding of how Courts operate in Europe. Our approach is inspired by a review of these procedures in most important EU patent jurisdictions which is presented in a companion paper.[5] That paper makes several important observations: First, that Courts in the EU give effect to FRAND commitment in the sense that if a prospective licensee makes a royalty offer that can be considered to be FRAND no injunction will be granted. Second, Courts do not typically directly enforce a FRAND rate but rather determine whether a proposed rate can be considered to be FRAND or not (and hence determine what is not FRAND). Third, the Courts will only grant injunctions if they can be convinced that the prospective licensee is unwilling. The elements that will be sufficient for a Court to conclude that a prospective licensee is unwilling might differ across jurisdictions, but will typically involve an analysis of the negotiation between the licensee and patent holder. In our model we capture this standard by assuming that the Court will only grant an injunction if the prospective licensee has made two offers that the Court deems not to be adequate.

Our model allows for a relatively rich strategic behavior by the prospective licensee, who can determine his offers considering the trade off between an attractice royalty that may imposed by the Courts and the risk of being found unwilling (and injuncted) . In Shapiro's model, the issue hardly arises as the prospective licensee will either be confronted with an enforceable court ruling on FRAND terms (in the benchmark scenario) or a negotiation under the threat of an enforceable injunction.

We find that despite the availability of injunctions, the holder of a sufficiently weak patent will end up accepting below FRAND rates, in particular when litigation cost are high. This is an instance in which the prospective licensee is holding up the patent holder [6] We also find that the prospective licensee will sometimes prefer to litigate. Indeed, the holder of a sufficiently strong patent will always end up in litigation by rejecting offers below FRAND. This arises in particular when the prospective licensee has little to fear from being found unwilling, namely when the trial takes time (so that the threat of injunctions is less powerful), and when litigation costs are low. Thus, we find that hold up (royalties above the fair rate) as well as reverse hold up (royalties below the fair rate) may arise in equilibrium. As it would appear that in some jurisdictions the Courts are willing to also determine a FRAND rate directly, if asked by one of the parties to do so, we also analyze such a scenario. We find that liability for the cost of litigation that is allocated by the courts on the basis of the initial offers always give rise to reverse hold up in this case.

The paper is organized as follows: In section 2 we discuss the relevant literature. Next in section 3 we set out our main model. Sections 4 and 5 consider extensions in which, respectively, the prospective licensee makes a single offer and in which the Court determines the FRAND rate directly. Section 6 discusses the role of injunctions and section 7

[4] Farrell, *supra* note 3

[5] Camesasca et al., *supra* note 4

[6] Joe Farrell at the June 2011 FTC Workshop: See transcript of "Tools to Prevent Patent Hold Up," FTC Workshop on Intellectual Property Rights and Standard Setting (21 June 2011), p. 243-45, http://www.ftc.gov/opp/workshops/standards/transcript.pdf.

concludes.

II. LITERATURE REVIEW

Besides Shapiro(2010)[7] and Lemley and Shapiro (2007)[8] several other papers address the role of injunctions and damages in the allocation of profits between the patent holder and potential licensee and the effect of these on innovation incentives.

The paper by Denicolo et al. (2008)[9] considers the role of injunctions in a framework similar to that of Lemley and Shapiro (2007)[10]. The authors take into account the fact that Courts might make mistakes in setting a FRAND rate. They observe that when Courts impose a rate that falls short of the fair value, the patent holder has no option but to accept that rate. As a result the patent holder will be under-rewarded in such circumstances. By contrast, when the rate exceeds the fair value, the prospective licensee can renegotiate. The extent to which the prospective licence can improve on the rate imposed by Court depends on the extent of hold up; if the circumvention cost is limited, the prospective licensee will obtain a royalty rate that is close to the fair value. The authors thus conclude that a regime in which Courts impose a royalty rate will induce both under-reward (due to the asymmetric nature of Court rulings) and over-reward (due to hold up). Our model differs from theirs in important ways. We consider a setting in which the cost of redesign is prohibitively high for the prospective licensee, so there is no scope for renegotiation after a rate has been imposed by Court. We also explicitly model the court decision with respect to the rate that it imposes and the award of injunctions as a function of the offers of the prospective licensee.

Schankerman and Scotchmer (2001)[11] study the effects of damages and injunctions on royalty negotiations for a technology that is indispensable for a new product development. With respect to injunctions, these authors focus on strategic behavior by the patent holder. They show that the effectiveness of injunctions in transferring value to the patent holder depends on the earliest date that the infringement will be enjoined. A delay in the date when the infringement is enjoined improves the bargaining position of the patent holder. This is because in the model of Schankermann and Scotchmer the cost that the infringer sinks into development of its product increases as time passes. Thus, when the implementer willfully infringes, the patent holder can wait for the sunk cost of the potential licensee to build up and threaten injunctions later in order to improve its bargaining position. Conversely, if the infringement is enjoined very early, the infringer has sunk only a small part of his costs and the extent of the licensee hold-up is smaller. When the patent holder can delay the date at which the infringement is enjoined, and the prospective licensee anticipates that, the licensee would no longer willfully infringe. In that case the license will be negotiated early, before the potential licensee has sunk significant cost so that the extent of the hold-up may again decrease. Even though Schankerman and Scotchmer do not look at the optimal enforcement schemes from the point of view of social welfare (or consumer surplus), they find that the availability of injunctions may be socially preferable to a world in which only damages are available.

Our model differs from that of Schankerman and Scotchmer in important ways. Whereas Schankerman and Schotchmer assume that injunctions are always granted, we make the award of injunctions contingent on the offers that are made by the prospective licensee. Unlike Schankermann and Scotchmer who focus on the strategic behavior of the patent holder in its choice of delay before bringing the infringement action, our framework allows for an analysis of the licensee's strategic behavior during royalty negotiations in order to affect the courts damage awards as well as the possible award of injunctions. Our model also explicitly allows for the possibility that the patent is invalid or not infringed, unlike Shankermann and Scotchmer's, and we study the effect of these factors on the allocation of profits.

Another related paper is Ganglmair et al.(2012)[12]. This paper considers an innovator (the patent holder) who decides how much to invest in developing an innovation. There is a single manufacturer (the licensee) who can use that innovation and decides on the level of specific investment in product development. The authors compare alternative regimes for the determination of royalties. In the first regime, there is no prior commitment on the licensing fee and this results in hold up of the manufacturer, who as a consequence undertakes an inefficiently low level of

[7] Lamley and Shapiro, *supra* note 1

[8] Shapiro, *supra* note 2

[9] Vincenzo Denicolo, Damien Gerardin, Anne Layne-Farrar and Jorge A. Padilla *Revisiting injunctive relief: interpreting eBay in High-Tech industries with non practising patent holders,* Journal of Competition Law and Economics (2008): 571

[10] Lamley and Shapiro, *supra* note 1

[11] Schankerman, Mark, and Suzanne Scotchmer, *Damages and injunctions in protecting intellectual property* RAND Journal of Economics (2001): 199-220.

[12] Bernhard Ganglmair, Luke M. Froeb, and Gregory J. Werden, *Patent Hold-Up and Antitrust: How A Well-Intentioned Rule Could Retard Innovation*, The Journal of Industrial Economics 60, no. 2 (2012): 249-273.

specific investment. In the second regime, the innovator and the manufacturer consider an option-to-license contract such that the inventor and the manufacturer can agree on the license fee before the manufacturer invests; in this scenario, there is no hold up. In the third scenario, a FRAND commitment is enforced after the manufacturer invests through a court review. The Court imposes damages on the patent holder if there is a breach of commitment. In this last scenario, there is no hold up either but the level of investment undertaken by the innovator is inefficiently low relative to the level achieved in the second scenario involving a fixed fee ex ante.

The scenario of enforceable FRAND commitment is particularly relevant for our analysis. In Ganglmair et al. either the manufacturer or the innovator makes a royalty offer (but not both simultaneously), with some positive probability. If the offer has been made by the innovator, the manufacturer has the option of litigating. He will indeed use this option in case the royalty fee offered by the patent holder is very high. On the other hand, the innovator cannot bring the manufacturer to court for making an inadequate offer - in this model, the FRAND commitment can only be enforced in Court by the manufacturer. This asymmetry limits the surplus that the innovator can extract from the manufacturer, but not vice-versa. The authors thus find that the one-sided FRAND commitment solves the manufacturer hold up problem but does not solve the potential hold up of the innovator and can thus retard innovation. The commitment has an ambiguous effect on incremental welfare in comparison to the benchmark situation in which there is no prior commitment. In other words, the FRAND commitment solves the hold up problem at the expense of making the reverse hold up worse and welfare might be reduced overall.

Ganglmair et al. model the court's determination of whether the offer is FRAND in the same way as we do. However, there are a number of differences. Unlike Ganglmair et al., we explicitly model injunctions and their award as contingent on the licensee's offers made. Hence, there is a trade off that the licensee has to consider in making an offer in our model: a lower offer, if accepted by the Court, of course benefits the licensee, but it carries a higher risk that it is not accepted as FRAND. In the latter case such an offer triggers the award of damages or it even triggers an injunction. By contrast, in the model of Ganglmair et al., Courts always enforce the FRAND rate (as long as the innovator makes the offer). From that prospective, their model is equivalent to a model in which Courts would simply set the FRAND rate (irrespectively of the offers made during negotiations). In addition, besides the damages based on the chosen FRAND royalty rate, the court in our model also determines who bears the litigation costs. This is relevant (and potentially desirable) for SEPs whenever the cost of litigation is significant or the patent holder deals with an unwilling licensee. Because of the availability of injunctions and the court's determination of who pays for the costs of litigation on the basis of the licensee's binding offer in the course of negotiations, a low offer will induce the patent holder (innovator) not to accept it and to rather bring infringement action and seek an injunction. Thus, a potential licensee risks that a low offer will indeed trigger an injunction and there is scope for both hold up of the licensee and the patent holder. Overall, our analysis thus contribute to the literature by characterizing how the strength of the patent, the length of litigation and the level of litigation costs affect strategic behavior of the potential licensee and determine the equilibrium royalties and in particular whether a hold-up of the licensee or the patent holder will obtain in equilibrium.

III. MODEL

The model captures the main features of institutions surrounding SEPs, namely FRAND commitments within a standard setting organization as well as the Court procedures with respect to the award of injunctions, as we understand them in the main European jurisdictions. An important feature of the model is that the determination of the FRAND rate in any particular case is surrounded with uncertainty. Moreover, rarely will the Courts be asked determine a FRAND royalty directly. Rather, the Courts will determine whether particular rates that have been proposed by the parties comport with FRAND or not. In addition, we understand that the Courts make the award of injunctions contingent on evidence that the prospective licensee is unwilling. This will generally take the form of an assessment of the willingness of the licensee to take a FRAND license. If the potential licensee makes a credible offer or otherwise signals the willingness to negotiate, injunctions are unlikely to be granted. For instance, the so called Orange Book procedure in Germany stipulates that the prospective licensee can always avoid an injunction if he makes an offer that can be considered to be FRAND (and complies with some other procedural requirements, see Camesasca et al.(2012) for details).

We formalize this situation with a game in which the Court draws a benchmark rate from a symmetric distribution around the true FRAND defined over some reasonable domain of potential FRAND rates. The true FRAND is private knowledge of the parties but the draw is private knowledge of the Court. Any proposed rate that is above the benchmark will be considered to comport with FRAND. In contrast, proposals below that rate will be deemed as non FRAND. Hence an unbiased court might make both type I and type II errors. In our benchmark model, we also allow the potential licensee to improve its offer if and once the Court has rejected the first offer as not being FRAND. We have observed such a sequence of events in one of the recent disputes between two large mobile handset manufacturers in Germany (see Camesasca et al.(2012)[13]) for details.

In our model, the court procedure also involves an initial assessment of the validity of the patent, in line with what most courts would undertake. This is however not what the German courts would typically do: in Germany the proceedings for nullity and determination of FRAND rate are separate. However, in a FRAND determination proceeding, the court would take account of an argument that a parallel nullity proceeding was launched, perhaps in a different jurisdiction. If the finding of invalidity of the patent is likely, the Court might defer deliberations of FRAND.

III.A The sequence of moves and events

After the licensee makes a binding offer, the patent holder either accepts or rejects it. If the patent holder accepts the offer, the game terminates. The licensee pays a running royalty for the patent until the end of the product life. We normalize this length to 1.

In case the patent holder does not accept the licensee's offer, litigation begins. Both the patent holder and the licensee pay their litigation costs. After a time period T, the court declares whether the patent is valid or not (the patent is expected to be valid with probability γ). If the patent is declared invalid, the patent holder pays damages for the litigation cost of the licensee and the game terminates. If the patent is valid, the Court draws a benchmark FRAND rate.

To simplify our exposition, in the most of the analysis we assume that this draw is made from a uniform distribution with expected value being the true FRAND. The Court compares the value of its draw to the offer that the licensee has made and if the latter is higher, it imposes licensing at the rate the licensee has offered. In this case, the patent holder is deemed to have wrongfully sought injunction against a willing licensee and it has to pay the licensee's litigation costs. At the same time, the licensee has to compensate the patent holder for the use of its patented technology during the Court proceedings.

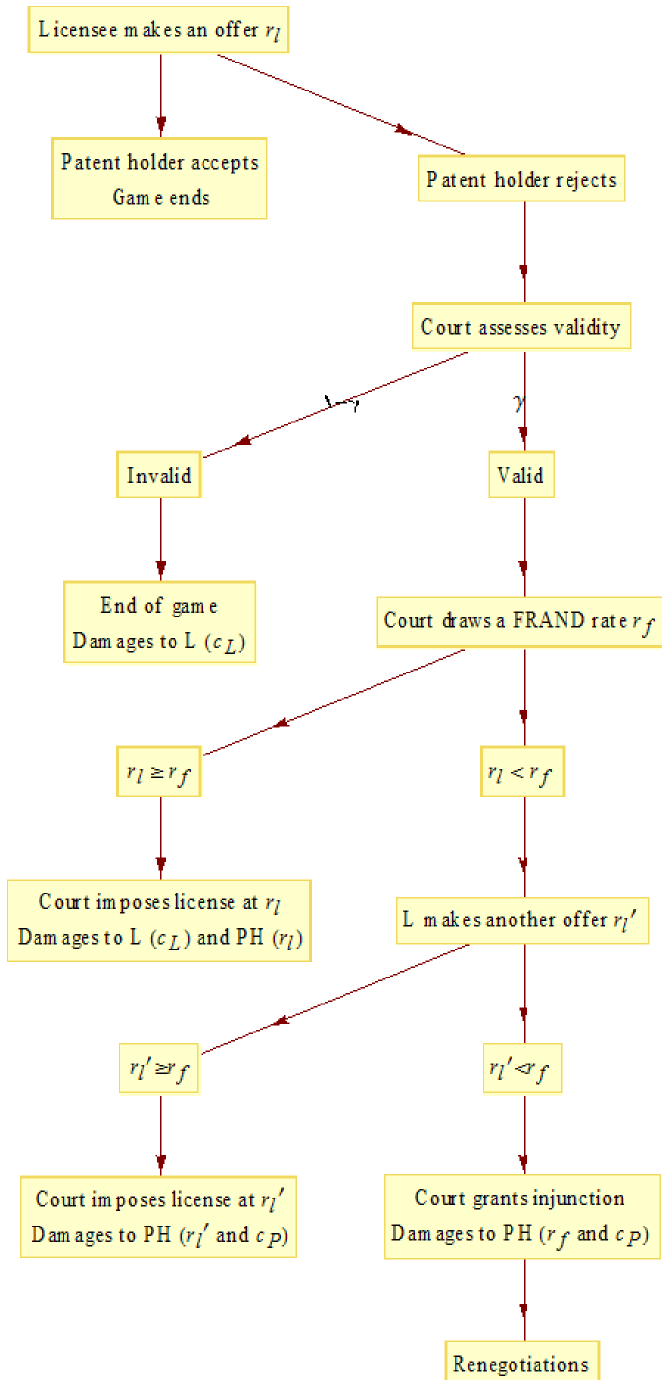
If the benchmark offer is higher than the rate proposed by the licensee, the patent holder is not deemed to have sought an injunction against a willing licensee. The licensee will thus have to compensate the patent holder for its litigation costs. Yet, before granting an injunction, the Court allows the licensee to improve its offer once. If the improved offer is deemed to comport with FRAND, the court imposes the license on the terms of the improved offer which applies both for the past and future use of the patented technology. However, if the improved offered rate is not higher that the court's benchmark rate, an injunction is granted and the licensee must pay damages at the benchmark rate for the past use of patented technology.

We conservatively assume that after the award of an injunction, all the bargaining power is with the patent holder who can thus extract the whole surplus from the licensee. In what follows, we first derive the subgame perfect equilibrium. We find that in some parameter range, the prospective licensee is better off inducing litigation and in some other range will prefer to make an offer that the patent holder will accept. Lastly, we characterize the equilibrium royalty rates relative to the FRAND level and thereby identifies the circumstance in which there will be respectively hold up (above FRAND rates) and reverse hold up (below FRAND rates).

The sequence of events is depicted in Figure 1.

[13] Camesasca et al., *supra* note 4

Figure 1: Sequence of events.



III.B Subgame perfect equilibrium

1. Final renegotiations

We start solving the model at the final stage for the case in which the injunctions have been granted. The continuation payoffs for the licensee (L henceforth) and the patentee (P henceforth), respectively, are:

$$\pi_L^F = 0,$$

$$\pi_P^F = (1-T)mX.$$

We give all the bargaining power to the patentee after he has obtained an injunction. Thus the licensee obtains zero profits. In P's payoff, the term $(1-T)$ represents the remaining lifetime of the product or technology. mX are the revenues that the product commands in the downstream market, net of all costs (X is exogenous demanded quantity, m is per-unit margin for products incorporating the patent).

2. The second court decision

Court finds that the second offer remains below its assessment of the FRAND rate $r_f > r_l'$, The payoffs are

$$\pi_L^{3In} = \pi_L^F - TXr_f - c_P,$$

$$\pi_P^{3In} = \pi_P^F + TXr_f + c_P.$$

The Court imposes its own assessment of the FRAND rate for the period of the trial (damages), grants an injunction and orders L to reimburse P for the legal cost that it has incurred.

Court accepts the second offer as FRAND. $r_l' \geq r_f$ The payoffs are

$$\pi_L^{3NI} = (1-T)mX - Xr_l' - c_P,$$

$$\pi_P^{3NI} = Xr_l' + c_P.$$

In this case L pays the litigation costs because the first offer r_l was not FRAND. The court imposes the second offer as remuneration for the period of the trial (damages). Game ends.

L makes the offer r_l'. L observes that its offer was judged by the court as not FRAND and updates her beliefs about the distribution of the court determined FRAND rate. This results in a new conditional probability distribution

$$f(x|x > r_l) = \frac{f(x)}{1-F(r_l)}.$$

The expected continuation payoff of L from setting the rate at $x \in [r_l, 1]$ is

$$\frac{F(x)-F(r_l)}{1-F(r_l)}\pi_L^{3NI} + \frac{1}{1-F(r_l)}\int_x^1 \pi_L^{3In}(a)dF(a),$$

where the first term stands for the payoff in case of acknowledgment of x as FRAND; the second term describes the payoff in case x was judged below FRAND.

First order necessary condition for an interior maximum of this program can be simplified to obtain

$$(1-T)(m-r_l') = \frac{F(r_l')-F(r_l)}{f(r_l')}.$$

Assuming the uniform distribution of FRAND rate over the unit interval, this condition can be rewritten as

$$r_l' = \frac{1}{2-T}((1-T)m + r_l). \quad (1)$$

It can be verified that the second order condition holds in case of uniform distribution.

Similarly, P's expected continuation payoff at rate r_l' is

$$\pi_P^{2A} = \frac{1}{1-F(r_l)}\left[\left(F(r_l') - F(r_l)\right)\pi_P^{3NI} + \int_{r_l'}^{1} \pi_P^{3In}(a)dF(a)\right].$$

3. *The first court decision*

Court accepts the first offer as FRAND, $r_l \geq r_f$. The payoffs are

$$\pi_L^{2NI} = (1-T)mX - Xr_l + c_L,$$

$$\pi_P^{2NI} = Xr_l - c_L.$$

The litigation costs are borne by P, because it should have accepted the offer at r_l. P gets the royalty Xr_l, whereas L obtains the benefit $(1-T)mX$ from the patented feature on the market.

Court draws the rate r_f after having ruled the patent valid. The probability that the offer was above FRAND is $F(r_l)$, so the expected continuation payoffs of the players are

$$\pi_L^1 = F(r_l)\pi_L^{2NI} + (1 - F(r_l))\pi_L^{2A},$$

$$\pi_P^1 = F(r_l)\pi_P^{2NI} + (1 - F(r_l))\pi_P^{2A}.$$

Court rules the patent invalid. The payoffs are

$$\pi_L^{inv} = (1-T)mX + c_L,$$

$$\pi_P^{inv} = -c_L.$$

P pays litigation costs, because it should not have claimed the patent. The benefit from the patented feature $(1 - T)mX$ is retained by L completely.

Patentee rejects the offer r_l - the litigation is initiated.

The payoffs are

$$\pi_L^0 = \gamma\pi_L^1 + (1-\gamma)\pi_L^{inv} + TmX - c_L,$$

$$\pi_P^0 = \gamma\pi_P^1 + (1-\gamma)\pi_P^{inv} - c_P.$$

Each player incurs its own litigation cost at this point. The expectation is formed over the event of confirming and rejecting validity of the patent. L's payoff has an additional term TmX reflecting the fact that it is active in the market, using the patented feature during the litigation period

Patentee accepts the offer r_l

The payoffs are

$$\pi_L^a = X(m - r_l),$$

$$\pi_P^a = r_l X.$$

P chooses not to litigate - there is no litigation cost. L pays royalty $r_l X$ to P while it obtains the benefit mX.

4. Licensee chooses the optimal take-it-or-leave it offer

In order to characterize the optimal offer, we proceed as follows. Firstly, we derive the optimal offer under the condition that it would be rejected and discuss its determinants (Lemma 1). Second, we derive the optimal offer under the constraint that it would be accepted by the patent holder and discuss its determinants (Lemma 2). In the parameter range for which the best offer that would be accepted exceeds the optimal offer under the condition that it would be rejected, we find that prospective licensee will sometimes prefer to trigger litigation (i.e. will select the optimal offer derived under the condition that it is rejected) and characterize the circumstances in which litigation is preferred. In the opposite case we show that the prospective licensee will never want to litigate. He will always propose the lowest rate that would be accepted by the patent holder. Henceforth we assume $m = 1$ implying that the upper bound of the distribution of the benchmark rate is equal to the net margin m.

The optimal offer under the condition that it is rejected. The optimal offer to make under the assumption that it will be rejected is the one that maximizes the payoff π_L^0. Under the uniform distribution, the first order condition for interior maximum is

$$\gamma\big((1-T)Xm - 2Xr_l + c_L\big) + \gamma \times$$
$$\times \left[X\left[r_l' + r_l\frac{dr_l'}{dr_l} - (1-T)\left(1 - \frac{dr_l'}{dr_l}\right) + 2\frac{T-2}{2}r_l'\frac{dr_l'}{dr_l}\right] + c_P\right] = 0,$$

where

$$\frac{dr_l'}{dr_l} = \frac{1}{2-T}.$$

The second order condition is satisfied.

Substituting, we obtain the optimal offer for L to make given that it will be rejected is

$$r_l^{rej} = \frac{(2-T)C + 1 - T}{3 - 2T}, \tag{2}$$

$$C := \frac{c_L + c_P}{X}.$$

Here, C is defined as the joint litigation costs of P and L normalized by the demand X. The following Lemma characterizes how our parameters affect the offer chosen by L to be rejected by P:

Lemma 1 *The strength of the patent γ has no effect on the offer r_l^{rej}. Litigation costs C increase the optimal offer that is going to be rejected. The effect of litigation time is negative, if gains from patented feature 1 are larger then litigation costs C.*

Proof: The first two statements follow trivially from the expression (2). To see why the last statement is true, define $t := 1 - T$, so

$$r_l^{rej} = \frac{C(1+t) + t}{1 + 2t}.$$

Differentiate this with respect to t to obtain

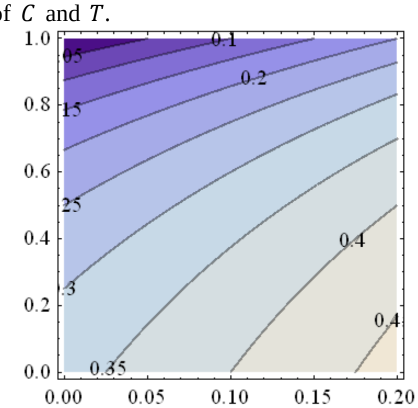
$$\frac{dr_l^{rej}}{dt} = \frac{1-C}{(1+2t)^2}$$

which is positive (and hence $\frac{dr_l^{rej}}{dT} < 0$), if $C < 1$. This is always satisfied in the interior solution, and follows from $r_l^{rej} < 1$.

It accords with intuition that the strength of the patent does not affect the offer that is assumed to trigger litigation. This arises because the payoff of the prospective licensee is unaffected by the value of this offer when the patent is invalid. In these circumstances, his legal cost is simply reimbursed by the patent holder. Accordingly, the prospective licensee defines his optimal offer as if the patent was valid. It is also intuitive that higher litigation costs increase the offer of the prospective licensee; in those circumstances, it will be more attractive to increase the odds of being considered FRAND, thereby avoiding litigation cost. Finally, as the litigation takes longer, the more remote is the prospect of an injunction. This makes it more attractive to increase the odds of being considered unwilling.

Figure 2 depicts the contours of the offer made by the licensee conditionally on it being rejected as a function of C and T. On the horizontal axis we depict C and on the vertical axis T. It is clear by looking at the contours that the optimal offer, conditional on being rejected increases in C and decreases in T.

Figure 2: Contours of r^{rej} in the space of C and T.



Note that the offer is bounded from above by 1 (since FRAND rates belong to the unit interval). In what follows we focus on interior solutions and accordingly we restrict the parameters as follows:

$$C < 1.$$

Optimal offer under the constraint that it is accepted by the patent holder. The optimal " accept" offer is the lowest offer x in the admissible range that equalizes the payoffs of the patent holder with and without litigation[14]:

$$xX = \pi_P^0(x). \tag{3}$$

Using the expression for the payoff of the patent holder derived above and rearranging, we obtain:

$$r_l^{acc} = \frac{T-2}{2T-3}\left(\frac{1-T}{2-T} + \frac{1}{\gamma} + C - \sqrt{\left(\frac{1-T}{2-T} + \frac{1}{\gamma} + C\right)^2 - 4\frac{2T-3}{2T-4}\left(\frac{2T-3}{2T-4} - \frac{1-\gamma}{\gamma}C\right)}\right). \tag{4}$$

We show in the appendix that a real solution always exists. In addition, we focus as before on interior solutions. As shown in the appendix, this implies the following constraint on parameters:

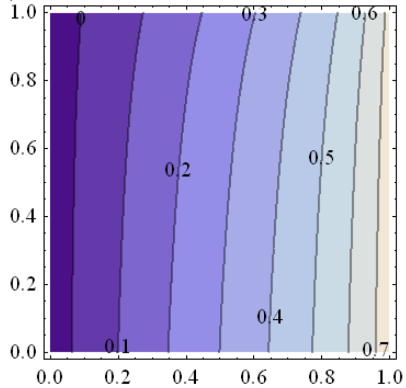
[14] As the expression below is a quadratic equation, there will be two rates satisfying it. But the higher rate will obviously not be offered by L.

$$\frac{1-\gamma}{\gamma}C < \frac{2T-3}{2T-4}.$$

Moreover,the constraint that the offer that would be rejected is interior r_l^{rej} also implies $C < 1$. It is easy to check that these constraints can be met simultaneously.

Figure 3 depicts the contours of the optimal offer made by the licensee such that it induces the patent holder to accept it in the space of γ (on the horizontal axis) and T (on the vertical axis). We fix $C = 0.05$. This graph reveals the offer is increasing in γ and is decreasing in T. The acceptance offer is further characterized in the following lemma.

Figure 3: Contours of r^{acc} in the space of γ and T.



Lemma 2 *The strength of the patent γ increases the offer r_l^{acc} that would be accepted. Litigation costs C and the length of litigation T decrease this offer.*

Proof:
See appendix.

We observe that litigation costs reduce the offer of the licensee. This arises because litigation costs reduce the litigation payoff of the patent holder, so that a lower offer will suffice to induce him to avoid litigation. It is also intuitive that the stronger the patent, the higher is the offer that the licensee has to make as the patent holder obtains a higher litigation payoff with a stronger patent. The litigation time also reduces the litigation payoff of the patent holder, as late injunctions are less powerful. Accordingly, he will be willing to accept a lower offer from the prospective licensee.

The choice of whether to induce litigation. The rejection and acceptance offers can now be compared to characterize the optimal choice of the licensee. When $r_l^{rej} \leq r_l^{acc}$, so that the optimal offer derived under the condition that it would be rejected is lower than the minimum offer that will induce the patent holder to accept, the licensee will choose r_l^{rej} if he wants rejection, and r_l^{acc} if he wants to induce litigation. If $r_l^{rej} > r_l^{acc}$, then r_l^{rej} is not valid as an optimal rejection offer, because it would not be rejected.

As the rejection payoff achieves its unconstrained maximum at r_l^{rej}, it is increasing for $r_l < r_l^{rej}$. Taking into account the constraint $r_l < r_l^{acc}$, the (constrained) maximum is achieved at $r_l = r_l^{acc}$, so that L chooses the same offer for acceptance or rejection in this case.[15]

In the range of parameters for which $r_l^{acc} > r_l^{rej}$, the offer r_l^{rej} will indeed induce litigation. This happens whenever

[15] We could also say that the optimal offer that induces litigation is $r_l^{acc} - \varepsilon$, where $\varepsilon > 0$ is arbitrarily small.

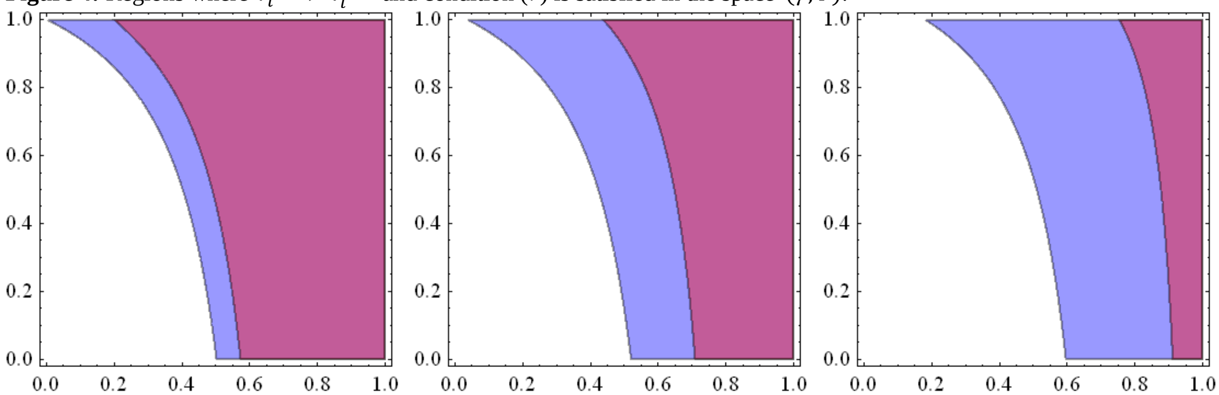
$$-2 - 2C^2\gamma - 10C + 2T + 4\gamma + 4C\gamma - 3T\gamma + C^2T\gamma + 6CT - 2CT\gamma > 0 \tag{5}$$

For very low costs $C \to 0$ this becomes

$$-2 + 2T + 4\gamma - 3T\gamma > 0,$$

so for low γ the inequality will not be satisfied, whereas for high γ it will be satisfied.

Figure 4: Regions where $r_l^{acc} > r_l^{rej}$ and condition (7) is satisfied in the space (γ, T).



We illustrate the region where $r_l^{acc} > r_l^{rej}$ in the space of (γ, T), for $C = 0.001$ (left), $C = 0.01$ (middle) and $C = 0.05$ (right) in Figure 1. The blue region represent the relevant set of parameter values (the red region represents another condition discussed later and is overlaid over the blue region). γ and T are respectively on the horizontal axis and vertical axis. From the figure it is clear that the region in which the accept offer exceeds the reject offer increases as C decreases. The lower the cost of litigation, the lower is the optimal rejection offer (see Lemma 1) as the licensee is more inclined to take the risk that he will have to pay the litigation costs (in case the court rules that the offer is below FRAND and the patent is considered valid). At the same time, with lower litigation costs, the offer that would be accepted increases (Lemma 2). As the patent holder's incentives to avoid litigation decrease with lower litigation costs he must be offered a higher rate to reject this option. Hence, since the acceptance offer increases and the rejection offer decreases (with lower litigation cost), the parameter space for which the acceptance offer is greater than the rejection offer will expand.

When the time in court is short, the licensee will want to make a high reject offer but the payoff from litigation for the patent holder is also relatively high (the threat of injunctions is powerful) and he is more likely to indeed reject the high offer of the licensee (the acceptance offer increases). However, the former effect is stronger than the latter and overall, as the time in court decreases, the parameter range for which the acceptance offer is higher than the reject offer will shrink. Finally, when the patent is likely to be invalid, the payoff from litigation decreases for the patent holder and, all else equal, he is more likely to accept an offer from the licensee (the acceptance offer decreases). Accordingly, the range of parameters for which the acceptance offer exceeds the rejection offer shrinks.

5. *Equilibrium payoffs*

We can summarize the payoffs of L in both cases as follows :

a) rejection payoff:

$$\frac{R_L}{\gamma X} = \frac{1}{\gamma} - C(1-a) + \frac{2(T-1)a + (3-2T)(a^2+1)}{2(T-2)}, \tag{6}$$

where

$$a := \min\{r_l^{rej}, r_l^{acc}\}.$$

b) the acceptance payoff:

$$\frac{A_L}{\gamma X} = \frac{1}{\gamma}(1 - r_l^{acc})$$

The following proposition describes how the parameters of the model affect the choice of equilibrium with (rejection offer) or without litigation (acceptance offer) for the case when unconstrained maximization of rejection payoff is not feasible.

Proposition 1 *When condition (5) is not satisfied, L chooses to post an offer that will be accepted.*

Proof: See appendix.

The proof of the proposition reveals that the payoff of the prospective licensee when he induces litigation with an offer arbitrarily close to the acceptance offer is lower than his payoff if he uses the acceptance offer by an amount which is equal to the litigation cost (and hence, he will prefer to avoid litigation). This result accords with intuition; indeed, by marginally increasing his offer (so that it is accepted), the licensee can save the entire litigation cost. Since he is making a take it or leave it offer to the patent holder, it accords with intuition that he should be in position to extract the entire increase in surplus associated with this marginal change.

When $r_l^{rej} < r_l^{acc}$ (condition (5) is satisfied), the rejection offer is below the acceptance offer. As discussed above, this will typically arise when litigation costs are relatively unimportant, when the time in court is long and when the patent is strong. Litigation in these circumstances may be preferred. In order to characterize whether the optimal choice of the licensee is to litigate or not in this range, we simply compare the payoff of L from litigation π_L^0 with her agreement payoff π_L^a at corresponding rates. We obtain the corresponding condition:

$$-\frac{1}{4T-6}(3T - 4 + (2T - 4)C + (2 - T)C^2) + \frac{r_l^{acc}}{\gamma} > 0 \tag{7}$$

Evaluating this expression, we find that for $r_l^{rej} < r_l^{acc}$ litigation is not always preferred. We depict this in the space of parameters (γ, T) for different levels of costs of litigation in Figure 1. Litigation is preferred in the range depicted with the red region of the figures. As the costs of litigation decrease the two regions become more and more closely aligned and as $C \to 0$ they are identical and thus the licensee induces litigation whenever $r_l^{rej} < r_l^{acc}$.

To sum up, we find that the prospective licensee will make an offer that the patent holder will accept for some parameters but will prefer to induce litigation typically when the trial time is long (the threat of injunctions is less powerful), when the patent is relatively strong (making an offer that would be accepted is too costly and it is preferable to take the risk of being considered unwilling) and when the cost of litigation is low.

The intuition behind this result can be expressed as follows: The prospective licensee essentially has to choose between a low initial offer that induces litigation and a higher "accept" offer that avoids the cost of litigation. He compares the payoff levels associated with these two alternatives. On the one hand, the payoff level associated with litigation at a low initial offer rate will be relatively high (and reach its maximum for low rates) when the prospective licensee is not concerned about the prospect of being found unwilling. This arises in particular when the proceedings are long so that the threat of injunctions is weak. At the same time, in order for the licensee to prefer litigation, the payoff level associated with the accept offer should be relatively low, or in other words, the accept offer should be relatively high. The accept offer will be high when the patent holder prefers to litigate with a high initial rate (as the accept rate makes the patent holder indifferent with litigation).

The patent holder will prefer to litigate with a high initial offer when the probability of validity is high. In those circumstances, a higher initial rate is likely to result into a higher FRAND rate imposed by the Court. By contrast, when the probability of validity is low, his payoff is not very sensitive to the initial offer, as it is likely that the patent holder will not get this FRAND rate in any event (as the patent is likely to be invalidated). Overall, it thus accords with intuition that litigation is preferred when the patent is likely to be valid and when the proceedings take a long time. In this model, litigation does take place in equilibrium because initial offers sometimes affect the payoffs of the prospective licensee and patent holder in a very asymmetric fashion. The saving in litigation cost that offering a rate which is accepted entails comes at too high a price for the prospective licensee.

III.C. Comparison of the initial offers with FRAND rate

The FRAND rate in our framework is exogenously set to equal $1/2$ for a valid patent. In this subsection we characterize the conditions for the equilibrium offer to be above (" hold-up" problem) or below FRAND rate (" reverse hold-up" problem). Since the patent is valid with probability γ, the expected FRAND rate is $\gamma/2$. We compare this with the equilibrium offer. In case the acceptance offer is used in equilibrium (and there is no litigation), the condition for hold-up is $r_l^{acc} > \gamma/2$. When the reject offer is below the acceptance offer and litigation occurs, the hold up condition becomes $r_l^{rej} > \gamma/2$. Then, the following corollary obtains directly from lemmas 1 and 2:

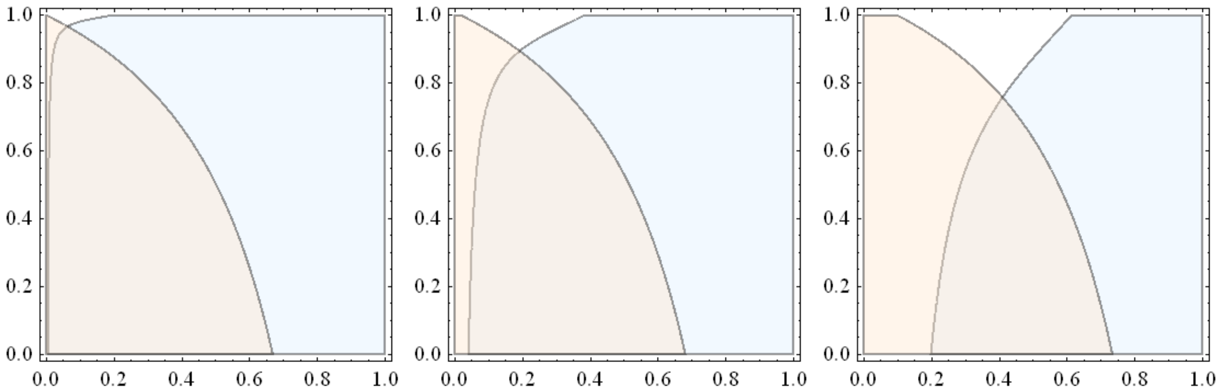
Corollary 1 *If the equilibrium of our game is characterized by litigation, then litigation costs C increase (decrease) the set of parameter values for which hold-up (reversed hold-up) problem arises; litigation time T has the opposite effect.*

If the equilibrium is characterized by no litigation, then costs C and litigation time T decrease (increase) the set of parameter values for which hold-up (reversed hold-up) problem arises; the strength of patent γ has the opposite effect.

Note that in the case the licensee finds it optimal to offer a royalty which will be accepted this is also the royalty which will be effectively applied. In this case, if the rate is below FRAND there is a reverse hold up problem generally. In the case the licensee finds it optimal to induce litigation, however, the rate that we compare to the FRAND rate here is not necessarily the rate that will effectively be applied after the court has reached a decision (and an injunction is still a threat). Thus, the effective rate, when it is optimal to induce litigation, might be above FRAND even when the initial rate that induces litigation is below FRAND. However, in such a case if the patent holder was forced to accept the offer (for example because of a threat of intervention of a competition authority), it would be held up.

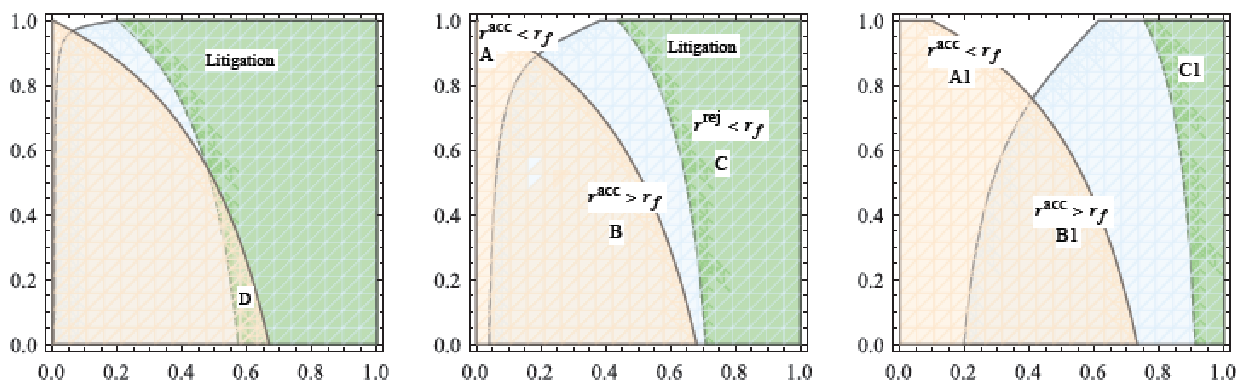
We represent the regions where the offer which induces acceptance and the initial offer which is optimal, conditional on it being rejected, are above FRAND in Figure 2. The orange-shaded area on the left side of each figure corresponds to the parameter range for which the rejection offer is above FRAND. The blue shaded area in the right-side of each figure corresponds to the range of parameters for which the acceptance rate is above FRAND. The figure in the left panel is for $C = 0.001$, in the middle for 0.01 and the one in the right panel for $C = 0.05$. We have γ and T respectively on the horizontal and vertical axis.

Figure 5: Regions where r_l^{acc} and r_l^{rej} are above FRAND in the space (γ, T).



In order to determine the regions in which the initial offer is below FRAND we superimpose, in Figure 3, the regions from Figure 2 on the regions in the space (γ, T) where the parties litigate. We do that both for different levels of costs ($C = 0.001$, $C = 0.01$ and $C = 0.05$).

Figure 6: Comparison of initial offer with FRAND rate in the space (γ, T).



In the three graphics of Figure 3, one can identify the regions for which initial offers are above or below the true FRAND. For combinations of γ and T in the green region, the licensee prefers to litigate - thus the licensee makes an offer that will be rejected so that the relevant offer to benchmark against FRAND is the one to be rejected.

We can identify the sub-regions of the litigation region where the initial offer that will be rejected is above FRAND. This will be the case when the green (litigation) region and the orange (reject offer above FRAND) region intersect. Such an intersection appears only in the left graphic of Figure 3 and is small (it is denoted by D). In all other cases the initial offer will be below FRAND (regions C and C1 in the middle and rightmost graphic). Note that the initial offer that will be rejected being below FRAND does not mean that the licensee will, in expectations obtain the license at costs below FRAND, but nevertheless, arguably, the patent holder was justified to reject it.

For combinations of γ and T outside the green region an offer will be made such that it is accepted. In that case the offer will almost always be above FRAND for very low cost of litigation in Figure 3. However, for somewhat higher cost of litigation it will be below FRAND for low γ and large T (the middle and the rightmost graphic, regions denoted by A and A1 respectively), but above FRAND for larger γ and low T (the middle and the rightmost graphic, regions denoted by B and B1 respectively), as can be seen from Figure 3).

We thus observe that both hold-up and reverse hold-up can result in this setting. A reverse hold up arises for wide ranges of the parameters. For example, when the parties do not litigate, the reverse hold up arises for a large set of γ and T when the cost of litigation are relatively high - the cost of litigation in this case favor the licensee. For intermediate values of validity and for relatively short litigation time, when litigation is not induced hold up occurs.

Clearly, the set of parameters where a reverse hold-up occurs is larger for large C, as can be seen from the figures.

We also observe that the effect of validity is not necessarily monotonous. For intermediate length of litigation (e.g. $T = 0.4$) and low costs of litigation (the leftmost panel of Figure 3) the patents that are extremely unlikely to be valid will be not be litigated and will achieve lower than FRAND rate. As their validity increase, quickly, the initial offer will be above FRAND - initially without litigation and then as entry offers which induce litigation. As the probability of the validity increases further, the entry rates into litigation will fall below FRAND again. This arises because the entry rate into litigation is independent of validity but the FRAND rate increases with validity. The situation with somewhat higher cost of litigation is slightly simpler as the initially offered rates are below FRAND for low γ and above FRAND for intermediate values of γ without inducing litigation. For high γ however, when the parties litigate, the initial offers are always below FRAND.

IV. ALTERNATIVE SPECIFICATION: THE LICENSE IS NOT ALLOWED TO IMPROVE THE OFFER

In this section, we consider a simplified version of the model in which the prospective licensee cannot improve his offer if it is rejected by the Court. The only thing that changes from our previous analysis is the expected continuation payoffs of the players when the court draws the rate r_f after having ruled the patent valid:

$$\pi_L^1 = F(r_l)\pi_L^{2NI} + \int_x^1 \pi_L^{3In}(a)dF(a),$$
$$\pi_P^1 = F(r_l)\pi_P^{2NI} + \int_x^1 \pi_P^{3In}(a)dF(a).$$

Following the same steps, we first derive the optimal rejection offer x that maximizes π_L^0:

$$\gamma\left(x\big((1-T)mX - Xx + c_L\big) + \int_x^1 (-TXa - c_P)da\right) + \\ +(1-\gamma)\big((1-T)mX + c_L\big) + TmX - c_L$$

Correspondingly, the first order condition for interior maximum implies

$$r_l^{rej} = \frac{M+C}{2-T}, \qquad (8)$$

where $M := (1-T)m$. Comparing the rejection rates across two scenarios, we obtain the following Lemma:

Lemma 3 *The offer to be rejected r_l^{rej} is higher when there is only one court decision.*

Proof: Directly comparing the two offers (4) and (8), we see that the difference between the offer in the scenario with two court decisions and the offer in the scenario with one court decision is

$$\frac{(2-T)C + M}{3-2T} - \frac{M+C}{2-T},$$

which can equivalently be rewritten as

$$(1-T)^2(C-1),$$

which is negative by interiority condition $C < 1$.

This result accords with intuition; when the licensee can make two offers, he finds it attractive to improve his initial offer if it is rejected. When there is a single offer, he quotes a higher offer from the outset. With two offers, he thus makes use of its additional degree of freedom (he could always mimic its decision in the scenario with one decision by simply quoting the same offer in the two stages).

Secondly, we derive the lowest offer x to be accepted by P:

$$xX = \gamma\left(x(Xx - c_L) + \int_x^1 ((1-T)mX + TXa + c_P)da\right) - (1-\gamma)c_L - c_P,$$

which can be solved for x to get the acceptance rate

$$r_l^{acc} = \frac{1}{2-T}\left(C + \frac{1}{\gamma} + M - \sqrt{\left(C + \frac{1}{\gamma} + M\right)^2 - 4\frac{2-T}{2}\left(M + \frac{T}{2} - C\frac{1-\gamma}{\gamma}\right)}\right). \tag{9}$$

The following Lemma characterizes how the rates in the two scenarios compare:

Lemma 4 *The offer to be accepted* r_l^{acc} *is higher when there is only one court decision.*

Proof: Directly comparing the two offers (10) and (9), we can numerically study the difference between the offer in the scenario with two court decisions and the offer in the scenario with one court decision in the compact set $[0,1]^3$ defined in the space of 3 parameters, γ, T and C. Analytically, one can study extrema of the function $f: [0,1]^3 \to \mathbb{R}$ that maps these parameters into the difference in the acceptance offers. Together with the border values of f; the extrema show that $\max_{[0,1]^3} f < 0$.

This lemma is also intuitive; the patent holder will be in better position in case of litigation when the prospective licensee can make a single offer. Accordingly, he has to be offered a higher rate to agree ex ante. Taken together, the lemmas also imply the hold up is more likely to occur with a single round of offer. The lemmas also allow us to formulate the following proposition:

Proposition 2 *With only one court decision, the licensee is worse-off and the patentee is better-off compared to the setup with two court decisions.*

Proof: See the appendix.

V. THE COURTS ANNOUNCING THE FRAND RATE AFTER THE FAILURE OF NEGOTIATIONS

In the third scenario we consider an institutional setup in which the court sets the FRAND rate directly after the negotiations break down. In this case, the courts decision effectively only determines which side has to pay the litigation costs. These costs are then the only potential source of holdup. The payoffs in this case can be easily written as

$$\pi_L^0 = (1-\gamma)Xm + \gamma X(m - r_f) - \gamma(1 - F(r_l))c_P + (1 - \gamma + \gamma F(r_l))c_L - c_L,$$
$$\pi_P^0 = \gamma X r_f + \gamma(1 - F(r_l))c_P - (1 - \gamma + \gamma F(r_l))c_L - c_P.$$

$$\pi_L^a = X(m - r_l),$$
$$\pi_P^a = r_l X.$$

The optimal offer to make under the assumption that it will be rejected is $r_l = 1$. In this case P always bears the cost of litigation. But such a high offer would never be rejected. Thus, the equilibrium offer is acceptance offer,

$$r^{acc} = \frac{\gamma r_f - (1-\gamma)C}{1 + \gamma C}.$$

Since the offer that is assessed by the court is made by the licensee, we expect the reverse hold-up to happen, which is indeed the case, since $r^{acc} < \gamma r_f$. In those circumstances, a high offer by the prospective licensee would shift the legal cost to the patent holder but would in all likelihood be accepted. Hence, the prospective licensee will reduce its offer to such a level that the patent holder is indifferent between accepting the offer and going to trial. This offer will be below the FRAND rate. To see why, assume that the patent is valid and that the prospective licensee makes a FRAND offer. At such offer the patent holder and prospective licensee expect that they will share the cost of the litigation equally (the probability that the Court makes a type I error balances the probability that the Court makes a type II error for the FRAND rate). The patent holder would thus accept a lower rate, which avoids his share of expected legal costs. Conversely, it is easy to verify, if the patent holder makes the offer, there will be hold up in equilibrium.

VI. THE ROLE OF INJUNCTIONS

Here we verify a very intuitive result that in the absence of injunctions ($T = 1$) the licensee is better-off and the patent holder is worse-off than in the benchmark model. Correspondingly, the likelihood of hold-up problem arising is reduced and the likelihood of reversed hold-up is increased. As in the previous subsection, we proceed in two steps: first, we show that the rejection rate is lower when injunctions are not available; second, we show that the acceptance rate is lower without injunctions. We conclude that injunctions increase the equilibrium offer and this makes the patentee better off and the licensee worse off.

Lemma 5 *The offer to be rejected* r_l^{rej} *is higher when injunctions are available.*

Proof: Since the comparative statics result of lemma 1 holds everywhere on $T \in (0,1)$ and there are no discontinuities in the payoffs at 0 or 1, we can conclude that the reject offer is lowest for $T = 1$. This is exactly the case of no injunctions.

Lemma 6 *The offer to be accepted* r_l^{acc} *is higher when injunctions are available.*

Proof: Analogous to that of lemma 5 making use of lemma 2 rather than lemma 1.

Taken together, the lemmas above allow us to formulate the following proposition:

Proposition 3 *Without injunctions, the licensee is better-off and the patentee is worse-off.*

Proof:
See the appendix.

VII. CONCLUSION

The concerns of leading competition authorities that injunctions sought by SEP holders may enable them to extract royalties from licensees that can be qualified as " excessive" may not be well founded. Using a simple model which captures some relevant institutional features surrounding enforcement of SEPs, we have shown that hold up does not necessarily arise in equilibrium in such a setting. This is true even when the courts may grant permanent injunctions which completely exclude the licensee from the market. In fact, it would appear that the licensee may often engage in a reverse hold up.

When the court bases its assessment of whether an injunction is legitimate on the licensee's offer, the licensee has a powerful strategic tool. And this provides the licensee with a significant advantage in bargaining which always reduces the extent of the hold up that the patent holder has over the licensee, but sometimes even completely reverses the balance of power. Specifically, we find that despite the availability of injunctions, the holder of a sufficiently weak patent will end up accepting below FRAND rates, in particular when litigation cost are high.

We also find that the prospective licensee will sometimes prefer to litigate and the holder of a sufficiently strong patent will always end up in litigation by rejecting offers below FRAND. This arises in particular when the prospective licensee has little to fear from being found unwilling, namely when the trial takes time (so that the threat of injunctions is less powerful), and when litigation costs are low.

We also consider the possibility that the courts would set a FRAND rate directly, if asked by the licensee of patentee, rather than issuing injunctions. In that case, the only scope for hold up stems from the cost of litigation. But insofar as the courts determine who is the loser in litigation (and thus who pays for litigation cost) by comparing the best offer of the licensee with what the court considers to be FRAND royalty, it is the licensee, not the patentee who holds up its counterpart.

VIII. APPENDIX

VIII.A Existence of real solution

To show that a real solution exists we study the expression for r_l^{acc}:

$$r_l^{acc} = \frac{T-2}{2T-3}\left(\frac{1-T}{2-T} + \frac{1}{\gamma} + C - \right. \\ \left. - \sqrt{\left(\frac{1-T}{2-T} + \frac{1}{\gamma} + C\right)^2 - 4\frac{2T-3}{2T-4}\left(\frac{2T-3}{2T-4} - \frac{1-\gamma}{\gamma}C\right)}\right). \quad (10)$$

We thus need to verify the following

$$\left(\frac{1-T}{2-T} + \frac{1}{\gamma} + C\right)^2 \geq 4\frac{2T-3}{2T-4}\left(\frac{2T-3}{2T-4} - \frac{1-\gamma}{\gamma}C\right),$$

which can be equivalently rewritten as

$$-\frac{2}{\gamma}\frac{2T-3}{(2T-4)(T-2)}(3\gamma - 4C + 4C\gamma - 2T\gamma + 2CT - 2CT\gamma) \leq \\ \leq \frac{1}{\gamma^2(T-2)^2}(T - \gamma - 2C\gamma + T\gamma + CT\gamma - 2)^2$$

or

$$-(2T-3)\gamma(3\gamma - 4C + 4C\gamma - 2T\gamma + 2CT - 2CT\gamma) \leq \\ \leq (T - \gamma - 2C\gamma + T\gamma + CT\gamma - 2)^2.$$

Squaring and collecting terms, we get

$$-(T-2)(T - 2\gamma - 2C^2\gamma^2 - 10C\gamma + 2T\gamma + 4\gamma^2 + 4C\gamma^2 - \\ -3T\gamma^2 + C^2T\gamma^2 + 6CT\gamma - 2CT\gamma^2 - 2) \leq 0$$

or

$$T - 2\gamma - 2C^2\gamma^2 - 10C\gamma + 2T\gamma + 4\gamma^2 + 4C\gamma^2 - 3T\gamma^2 + C^2T\gamma^2 + 6CT\gamma - 2CT\gamma^2 - 2 \leq 0.$$

Collecting terms again, we have

$$0 \leq (2-T)(1 + C^2\gamma^2 - 2C\gamma^2) + (3T-4)\gamma^2 + 2(1-T)\gamma + 2(5-3T)C\gamma$$

The term $-2C\gamma^2$ is dominated by $2(5-3T)C\gamma$; $(3T-4)\gamma^2$ is dominated by $2(1-T)\gamma$ and $2-T$. Thus, there is always a real solution.

The interiority dictates $0 < r_l^{acc} < 1$ or

$$\frac{1-T}{2-T}+\frac{1}{\gamma}+C>\sqrt{\left(\frac{1-T}{2-T}+\frac{1}{\gamma}+C\right)^2-4\frac{2T-3}{2T-4}\left(\frac{2T-3}{2T-4}-\frac{1-\gamma}{\gamma}C\right)},$$
$$\frac{1-T}{2-T}+\frac{1}{\gamma}+C-\sqrt{\left(\frac{1-T}{2-T}+\frac{1}{\gamma}+C\right)^2-4\frac{2T-3}{2T-4}\left(\frac{2T-3}{2T-4}-\frac{1-\gamma}{\gamma}C\right)}<\frac{2T-3}{T-2},$$

together with the previous condition,

$$C<1.$$

Simplifying, we have

$$\left(\frac{1-T}{2-T}+\frac{1}{\gamma}+C\right)^2>\left(\frac{1-T}{2-T}+\frac{1}{\gamma}+C\right)^2-4\frac{2T-3}{2T-4}\left(\frac{2T-3}{2T-4}-\frac{1-\gamma}{\gamma}C\right),$$
$$\left(\frac{1-T}{2-T}+\frac{1}{\gamma}+C-\frac{2T-3}{T-2}\right)^2<\left(\frac{1-T}{2-T}+\frac{1}{\gamma}+C\right)^2-4\frac{2T-3}{2T-4}\left(\frac{2T-3}{2T-4}-\frac{1-\gamma}{\gamma}C\right),$$

or

$$0>-4\frac{2T-3}{2T-4}\left(\frac{2T-3}{2T-4}-\frac{1-\gamma}{\gamma}C\right),$$
$$1-2\left(\frac{1-T}{2-T}+\frac{1}{\gamma}+C\right)\frac{2T-3}{T-2}<-4\frac{2T-3}{2T-4}\left(\frac{2T-3}{2T-4}-\frac{1-\gamma}{\gamma}C\right).$$

After another round of simplifications we get the condition in the text:

$$\tfrac{1-\gamma}{\gamma}C<\tfrac{2T-3}{2T-4}.$$

VIII.B Proof of Lemma 2

$$x=\tfrac{T-2}{2T-3}\left(\tfrac{1-T}{2-T}+\tfrac{1}{\gamma}+C\text{-}\right.$$
$$\left.-\sqrt{\left(\frac{1-T}{2-T}+\frac{1}{\gamma}+C\right)^2-4\frac{2T-3}{2T-4}\left(\frac{2T-3}{2T-4}-\frac{1-\gamma}{\gamma}C\right)}\right)$$

Implicitly differentiating the indifference condition (3), we get

$$D\frac{dr_l^{acc}}{d\gamma}=(2-T)\frac{1}{\gamma^2}(r_l^{acc}+C),$$
$$D=\frac{\partial\left(xX-\pi_P^0(x)\right)}{\partial x}\Big|_{x=r_l^{acc}}.$$

Since for the relevant solution the higher rate would be accepted by P, whereas the lower one would not be, $xX-\pi_P^0(x)$ switches its sign from negative to positive and hence is increasing at r^{acc}, so $D>0$. It immediately follows that $\frac{dr_l^{acc}}{d\gamma}>0$.

For the effect of C, directly differentiating the expression (10), we get

$$\frac{dx}{dC}=\frac{T-2}{2T-3}\left(1-\frac{2\left(\frac{1-T}{2-T}+\frac{1}{\gamma}+C\right)+4\frac{2T-3}{2T-4}\frac{1-\gamma}{\gamma}}{2\sqrt{\left(\frac{1-T}{2-T}+\frac{1}{\gamma}+C\right)^2-4\frac{2T-3}{2T-4}\left(\frac{2T-3}{2T-4}-\frac{1-\gamma}{\gamma}C\right)}}\right)$$

Comparing the nominator and denominator in the brackets, we establish, that the nominator is greater:

$$2\left(\frac{1-T}{2-T}+\frac{1}{\gamma}+C\right)+4\frac{2T-3}{2T-4}\frac{1-\gamma}{\gamma}>$$
$$>2\sqrt{\left(\frac{1-T}{2-T}+\frac{1}{\gamma}+C\right)^2-4\frac{2T-3}{2T-4}\left(\frac{2T-3}{2T-4}-\frac{1-\gamma}{\gamma}C\right)}$$

$$4\frac{2T-3}{2T-4}\frac{1-\gamma}{\gamma}\left(\frac{1-T}{2-T}+\frac{1}{\gamma}+C\right)+\left(2\frac{2T-3}{2T-4}\frac{1-\gamma}{\gamma}\right)^2$$
$$+4\frac{2T-3}{2T-4}\left(\frac{2T-3}{2T-4}-\frac{1-\gamma}{\gamma}C\right)>0$$

$$4\frac{2T-3}{2T-4}\frac{1-\gamma}{\gamma}\left(\frac{1-T}{2-T}+\frac{1}{\gamma}\right)+\left(2\frac{2T-3}{2T-4}\frac{1-\gamma}{\gamma}\right)^2+4\frac{2T-3}{2T-4}\frac{2T-3}{2T-4}>0$$

Hence the claim of the lemma, $dx/dC<0$.

Finally, to establish the effect of T, we directly differentiate the expression (10):

$$\frac{dx}{dT}=-\frac{1}{(2T-3)^2}+\left(\frac{1}{\gamma}+C\right)\frac{1}{(2T-3)^2}-$$
$$-\frac{-2\left(\frac{1-T}{2-T}+\frac{1}{\gamma}+C\right)\left(\frac{T-2}{2T-3}\right)^2\frac{1}{(2-T)^2}+2\left(\frac{1-T}{2-T}+\frac{1}{\gamma}+C\right)^2\left(\frac{T-2}{2T-3}\right)\frac{1}{(2T-3)^2}+2\frac{1-\gamma}{\gamma}C\frac{1}{(2T-3)^2}}{2\sqrt{\left(\frac{1-T}{2-T}+\frac{1}{\gamma}+C\right)^2\left(\frac{T-2}{2T-3}\right)^2-1+2\frac{1-\gamma}{\gamma}C\frac{T-2}{2T-3}}},$$

which has the same sign as

$$\left(\frac{1}{\gamma}+C-1\right)\sqrt{\left(\frac{1-T}{2-T}+\frac{1}{\gamma}+C\right)^2\left(\frac{T-2}{2T-3}\right)^2-1+2\frac{1-\gamma}{\gamma}C\frac{T-2}{2T-3}}-$$
$$-\left(\frac{1-T}{2-T}+\frac{1}{\gamma}+C\right)^2\left(\frac{T-2}{2T-3}\right)+\frac{1-\gamma}{\gamma}C-\left(\frac{1-T}{2-T}+\frac{1}{\gamma}+C\right),$$

or

$$\left(\frac{1}{\gamma}+C-1\right)^2\left(\left(\frac{1-T}{2-T}+\frac{1}{\gamma}+C\right)^2\left(\frac{T-2}{2T-3}\right)^2-1+2\frac{1-\gamma}{\gamma}C\frac{T-2}{2T-3}\right)-$$
$$-\left(\left(\frac{1-T}{2-T}+\frac{1}{\gamma}+C\right)^2\left(\frac{T-2}{2T-3}\right)+\frac{1-\gamma}{\gamma}C-\left(\frac{1-T}{2-T}+\frac{1}{\gamma}+C\right)\right)^2,$$

which can be simplified to

$$-\frac{1}{\gamma^2}(C-\gamma+1)^2,$$

which is clearly negative. Hence the claim of the lemma, $dx/dT<0$.

VIII.C Proof of Proposition 1

The difference of the payoffs of L in cases of rejection and acceptance at the same rate r_l^{acc} is

$$\frac{1}{\gamma}-C(1-a)+\frac{2(T-1)a+(3-2T)(a^2+1)}{2(T-2)}-\frac{1}{\gamma}(1-a),$$

which can be rewritten as

$$-\frac{2T-3}{2(T-2)}a^2+\left(\frac{1}{\gamma}+\frac{T-1}{T-2}+C\right)a+\frac{3-2T}{2(T-2)}-C.$$

At the same time, the indifference condition for the P requires that

$$-\frac{2T-3}{2T-4}a^2+\left(\frac{1-T}{2-T}+\frac{1}{\gamma}+C\right)a-\frac{2T-3}{2T-4}+\frac{1-\gamma}{\gamma}C=0.$$

From the last two conditions, we can deduce that the acceptance payoff is always larger than the rejection payoff at the same rate r_l^{acc}, with difference being CX.

VIII.D Proof of Proposition 2

In case of agreement, the payoff of L is decreasing in the offer, the payoff of P is increasing in it. By lemma 3, the statement of proposition follows for the case of agreement. For the case of litigation, we directly compute the payoffs and compare them. For the benchmark case, from (1), (2) and (6) we get

$$\frac{R_L}{\gamma X}=\frac{1}{2T-3}\left(2+2C-\frac{3T}{2}-C^2-CT+\frac{C^2T}{2}\right)+\frac{1}{\gamma}. \tag{11}$$

Similarly, writing explicitly the expression for π_P^0, we get

$$\frac{R_P}{\gamma X}=r_lr_l+(r_l'-r_l)r_l'+(1-r_l')(1-T)+\frac{1}{2}(1-r_l'^2)T-\left(\frac{1}{\gamma}-1+r_l\right)C.$$

Again, substituting from (1) and (2) we have

$$\frac{R_P}{\gamma X}=-\frac{1}{2\gamma(2T-3)}(4\gamma-6C+4C\gamma-3T\gamma-2C^2\gamma+4CT-2CT\gamma+C^2T\gamma). \tag{12}$$

For the alternative specification, we get the payoff of L as

$$\frac{\pi_L^{alt}}{\gamma X}=\left(r_l(1-T-r_l)-\frac{1}{2}T(1-r_l^2)\right)+\frac{1}{\gamma}(1-T-\gamma(1-T)+T)-(1-r_l)C.$$

Substituting the royalty rate from (8), we get

$$\frac{\pi_L^{alt}}{\gamma X}=-\frac{1}{4\gamma-2T\gamma}(-\gamma C^2+2\gamma C+2T+3\gamma-2T\gamma-4). \tag{13}$$

The payoff of P is

$$\frac{\pi_P^{alt}}{\gamma X} = r_l^2 + (1 - r_l)(1 - T) + \frac{1 - r_l^2}{2} T - \left(\frac{1}{\gamma} - 1 + r_l\right) C.$$

Substituting the rate from (8), we get

$$\frac{\pi_P^{alt}}{\gamma X} = -\frac{1}{2\gamma(T-2)}(3\gamma - 4C + 2C\gamma - 2T\gamma - C^2\gamma + 2CT).$$

Finally, we simply take the difference of the payoffs in two scenarios:

$$\frac{\pi_L^{alt} - R_L}{\gamma X} = -\frac{1}{2}(C-1)^2 \frac{(T-1)^2}{2T^2 - 7T + 6} < 0,$$
$$\frac{\pi_P^{alt} - R_P}{\gamma X} = \frac{1}{2}(C-1)^2 \frac{(T-1)^2}{2T^2 - 7T + 6} > 0.$$

As can be seen, the statement of the proposition follows also for the case of litigation.

VIII.E Proof of Proposition 3

In case of agreement, the payoff of L is decreasing in the offer, the payoff of P is increasing in it. By lemma5, the statement of proposition follows for the case of agreement.

For the case of litigation, we directly perform comparative statics exercise with respect to T for the payoffs of P and L. Simply differentiating (11) and (12), we get

$$\frac{\partial R_L / \partial T}{\gamma X} = \frac{1}{2(2T-3)^2}(C-1)^2 > 0,$$
$$\frac{\partial R_P / \partial T}{\gamma X} = -\frac{1}{2(2T-3)^2}(C-1)^2 < 0.$$

Clearly then, the litigation payoff of L achieves its maximum at $T = 1$ and the litigation payoff of P achieves its minimum. The statement of the proposition follows.